**Supporting Information A**

This section is to show a detailed clarification associated with the dimensionless transformation. Dimensionless is the removal of physical dimensions from a system containing physical quantities through appropriate variable substitution, so that the dimension of the variables in the transformed system is 1. Considering that the use of dimensionless transformation in a complex model can greatly reduce the difficulty of derivation and application for the analytical solution, appropriate dimensionless is necessary. Otherwise, the general use of dimensionless transformation can also facilitate the comparison between models.

The proposed dimensionless method is applied to the governing equation, i.e. Equation (1), three boundary conditions, i.e. Equations (2), (3) and (4), and the Izbash’s equation, i.e. Equation (5).

$\frac{∂q}{∂r}+\frac{q}{r}=\frac{S}{b}\frac{∂s}{∂t}$ (1)

$s\left(r\rightarrow \infty , t\right)=0$ (2)

$\lim\_{r\to r\_{w}}2πbrq\left(r,t\right)=-Q+πr\_{c}^{2}\frac{∂s\_{w}\left(t\right)}{∂t}$ (3)

$s\left(r, 0\right)=0$(4)

$q=\left(K\frac{∂s}{∂r}\right)^{\frac{1}{n}}$ (5)

In order to ensure that the law of the main parameters, including $r$, $r\_{w}$,$r\_{c}$, $r\_{i}$, $s$, $s\_{w}$, $t$ and $K$, cannot be changed after dimensionless transformation, the above-mentioned parameters whose dimension is [L] can be converted by dividing by the thickness of the aquifer as following.

$r\_{D}=\frac{r}{b}$ (α1)

$r\_{cD}=\frac{r\_{c}}{b}$ (α2)

$r\_{wD}=\frac{r\_{w}}{b}$ (α3)

$r\_{iD}=\frac{r\_{i}}{b}$ (α4)

where $r\_{D}$, $r\_{cD}$, $r\_{wD}$ and $r\_{iD}$ are the dimensionless $r$, $r\_{c}$, $r\_{w}$ and $r\_{i}$, respectively.

Then, the dimensionless transformation of $s$ and $t$ depends on a common form in related research, which can be more easily compared with other pumping models (Feng et al., 2019). This transformation method is as follows.

$s\_{D}=\frac{4πK^{\frac{1}{n}}b}{Q}s$ (β1)

$s\_{wD}=\frac{4πK^{\frac{1}{n}}b}{Q}s\_{w}$ (β2)

$t\_{D}=\frac{K^{\frac{1}{n}}t}{Sb}$(β3)

where $s\_{D}$, $s\_{wD}$ and $t\_{D}$ are the dimensionless $s$, $s\_{w}$ and $t$, respectively.

Substituting the above dimensionless equations into Equations (1), (3) and (5), a suitable dimensionless method for $K$ can be obtained as

$k\_{D}=\frac{4πK^{\frac{1}{n}}b^{2}}{Q}$ (γ1)

And the dimensionless equation for $q$ can be given as

$q\_{D}=\frac{4πb^{2}}{Q}q$ (γ2)

where $k\_{D}$ and $q\_{D}$ are the dimensionless $K$ and $q$, respectively.

The dimensionless transformation relationships of Equations (α1) to (γ2) have been listed in Table 1 for presentation.